A Note on Arabic Numerals

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The following lines I like to use as a crashcourse on arabic numerals addressed to the (german) audience of my lectures[*]. Students attending, e.g., *Algorithms in Number Theory* usually enjoy a „pause“ filled with some remarks about the historical and cultural background of the numerals used to denote numbers. Sigrid Hunke, in her book *Allahs Sonne über dem Abendland* (1960, Allah’s Sun Above the Occident), has put the essence of it into the words: „The arabic numerals have conquered the western world and continued playing a leading part in science, technology, economy, and traffic of all civilized people and for all times.“ (transl. A.S.)

Speaking of the numerals, of course, implies the use of the positional system based upon repeatedly combining units to bundles of fixed size, e.g,, ten. It is well known that such systems already had been used in China two thousand years ago. The number that appears as the result at a given stage counts those bundles (or units) which don’t fill a bundle at the next stage. If there remains any one could leave the place empty. However, it is more convenient to have a sign for emptiness: sunya — as Indian mathematicians called it to whom the ingenious invention of zero (in the nineth century) is ascribed. Robert Kaplan, in his brillant „natural history of zero“ makes this point emphatically by saying: „If you look at zero you see nothing; but look through it and you will see the world. For zero brings into the focus the great, organic sprawl of mathematics, and mathematics in turn the complex nature of things.“ (The Nothing that is, Penguin 1999, p.1)

As is well known the arabs not only recognized the great potential of this notation system but decisively contributed to its further development and proliferation towards the West. According to the possible cardinalities of the bundles in the decimal system ten characters are required for representation. Their form, of course, doesn’t matter. So, the common western digits actually differ from their eastern-style sisters:

٩	٨	٧	٦	٥	٤	٣	٢	١	٠
9	8	7	6	5	4	3	2	1	0

Europeans tend to have some minor difficulties. In eastern-style ‚four‘ looks like a ‚3‘ reflected, and ‚six‘ and ‚five‘ resemble ‚7‘ and ‚0‘ respectively. In arab everyday life both numeral styles are in use: e.g., eastern-style numerals serve for page numbering in books whereas the western-style numerals would be used in price-lists or number plates:

Leonardo of Pisa, outstanding mathematician of the western Middle Ages, had substantially contributed to this final success of the positional system. Nowadays Leonardo is better known

[*] A german version is to be found in the web: http://www.uni-flensburg.de/mathe/dzettel/0042/dz_0042.html

as Fibonacci (i.e. son of Bonacci, an influential merchant with connections to the court of Frederic II, emperor, savant and connaisseur of the islamic world). In his famous book *Liber Abaci* (Book of Computing) Fibonacci translated the arabic word for ‚0' (sifr) as „cephyrum". In italian, later, this became „zefero" finally transformed to „zero". The german word „Ziffer" (english: cipher) which has a more general meaning directly reflects its arabic origin.

Now, let's have a look at how a numeral consisting of several digits will be written in arabic. Perhaps due to the fact that arabic writing runs from right to left the word begins (at its right end) with the bundles of lowest order. Nevertheless, reading a numeral as, for instance, 3721 may proceed in both directions: one-and-twenty-and-seven hundred-and-three thousand, or reversely: three thousand-and-seven hundred-and-twenty-and-one.

The digit which denotes the number ‚one' deserves particular attention. It is both the first letter (Alif) of the arabic alphabet and symbol of the one and only God whose name „Allah" begins with an alif. The 112th sure of the Quran dedicated to this uniqueness inspired the german poet and orientalist Friedrich Rückert (1788-1866) to some extraordinary lines of mystical as well as mathematical poetry:

So wahr als aus der Eins die Zahlenreihe fließt,
So wahr aus einem Keim des Baumes Krone sprießt,
So wahr erkennst du, daß der ist einzig einer,
Aus welchem alles ist, und gleich ihm ewig keiner.

(Die Weisheit des Brahmanen, 1843)

The numbers all derive from number one,
The crowns of trees sprout from a single germ,
So you will see Him as the Only and the One,
The origin of all and like Him never one.

(Transl. A.S.)